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INNOVATION ABSTRACTS

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Using Mathematical Essays

This semester I "inherited" a (freshman) Calculus class that is small in number (10) but huge in talent. Recognizing such potential, I demanded that students display their problem-solving mathematics abilities beyond mere computations. One of the most fruitful assignments was the result of a test question: Write an essay to compare and contrast techniques of integration. I have chosen short excerpts from their essays. I would not be surprised if you read more of their work in future years. Enjoy!

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It is the plight of every Calculus student to discover, mainly through trial and error, the basic categories of integration common to all types of problems. By doing this, the student has a general form in mind, and by performing any number of legal substitutions or changes, can then reduce the original problem to a known form that he can solve. The two categories of integration, in my opinion, are the power rule and definition. All other strategies are merely methods to change the form of a problem to coincide with the power rule or definition categories. For example, problem 2 on this test is really the power rule in disguise. Trigonometric and u-substitutions are means of changing the problem into that form.

eg. $\int \sin^6 x^3 \cos x\, dx$

$= \int \sin^6 x(1 - \sin^2 x) \cos x\, dx$ trig sub

$= \int \sin^6 x \cos x\, dx - \int \sin^8 x \cos x\, dx$

$= \int u^6\, du - \int u^8\, du$ u-sub

POWER RULE

Problem 3, in the same way, is an example of definition and trig is a way of proving it.

eg. $\int \frac{dx}{x^2\sqrt{9 - x^2}}$ $\quad 1/9 \int CSC^2 \varnothing\, d\varnothing$

with $\varnothing = 3 \sin x$ as substitution.

Other methods of reducing a problem include completing the square, parts, partial fractions, and others that I have not learned. By utilizing one method or a combination of methods, every problem can be reduced to either power rule or definition.

—J. Clark

When evaluating integrals one should think of finding area. Integration is very helpful when finding the area under any given line, and formulas were generated to help us derive these answers. —E. Paynton

One should always keep in mind to simplify the problem at all times. Keep it simple. These (above) are the steps I use to break down an integration problem and solve it. Once the correct and easiest method of integration is found, the problem is half-solved.

—L. Anderson

For the most part we use our powerful tool of u-substitution to get the functions into correct forms.

—B. Miller

Knowing what the integral is, how to use u-substitution, and the power rule will help us through all integration.... Although there are many types of integration which have some obvious differences, they all have some fundamental similarities. All forms attempt to reach the same goal, make use of manipulation, and take careful work. If you choose the 'wrong' form it could take hours and get ugly.

—G. Abele

A majority of the integrals, in one way or another, use the power rule. This is the first form learned and usually easiest to solve. Most problems require a u-substitution to achieve this.... With simple u-substitutions, most problems can be simplified into forms to which tables and definitions will give an answer.

—M. Comeaux

Before chapter 9, integration was fairly straightforward.... Now some are even complex enough to the point where they must be looked up in a book.

—A. Teixiera

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Finally, if you cannot integrate by any method that was just discussed, we could approximate with left-handed end-point approximation, right-handed end-point approximation, trapezoidal approximation, or Simpson's rule. —J. Tarango

Numerical integration is very time-consuming, but quite simpler than other ways. All it requires is algebra. —L. Dongallo

In the beginning there was pre-chapter 9 integrals. You know, those 'simple' integrals that you can 'whip-out' with 'no problem'.... But then came the dreaded chapter 9 integrals and new techniques of integration. Let's recall the simple integral $\int e^x dx = e^x + c$. By simply placing an x in front of the e^x, you get a whole new problem. Now in comes one of our many heroes, integration by parts. Let's observe the battle:

"Evil Integral"	"Hero"
$\int xe^x\,dx$	$\int u\,dv = uv - \int v\,du$

Let $u = x \quad dv = e^x\,dx$

$$du = dx \qquad v = e^x$$

$$\begin{aligned} \int xe^x\,dx &= \int u\,dv \\ &= uv - \int v\,du \\ &= xe^x - \int e^x\,dx \\ &= xe^x - e^x + c \end{aligned}$$

The battle is over. Good prevailed over Evil....

Pre-chapter 9 and chapter 9 integrals have their differences as you have seen in the examples. Yet, u-substitutions are used in almost all of the integrals, maybe not right away...but somewhere in the middle, after things have been simplified.... Yes, the battle with integration seems to be getting harder and harder, but there is a new hero, Tables. His motto is, 'Look up the form, plug the values, and there's your answer.' So the quest continues to conquer integration, but especially his ruler, Calculus. —P. Quesada

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